

CONCORDIA UNIVERSITY
Department of Mathematics & Statistics

Course	Number	Section(s)
Mathematics	208/4	All

Examination	Date	Pages
Final	April 2007	3

FORMULAE:

$$A = P(1+i)^n, \quad A = Pe^{rt}, \quad FV = PMT \frac{(1+i)^n - 1}{i}, \quad PV = PMT \frac{1 - (1+i)^{-n}}{i}$$

Special Instructions:

- ▷ Answer all questions.
 - ▷ **Non-programmable calculators only** are permitted.
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MARKS

- [10] **1.** Let price p in dollars be a linear function of the demand as well as supply q (in hundreds) of T-shirts at a spring fair. The demands for 100 and 300 T-shirts yield prices of 4 and 8 dollars respectively. On the other hand, the supplies of 100 and 300 T-shirts yield prices of 10 and 8 dollars respectively.
- (A) Find the price p as a function of demand q (in hundreds), which is the demand equation.
- (B) Find the price p as a function of supply q (in hundreds), which is the supply equation.
- (C) Find the equilibrium price and demand (supply).
- (D) Graph these two price equations for $0 \leq q \leq 10$.
- [10] **2.** Solve for the unknown x in the following equations:
- (A) $272^{12x+987} = 272^{132x-453}$.
- (B) $(7\pi)^{3x^2-18x+152} = (7\pi)^{7x^2-6x-64}$.
- (C) $\log \sqrt[3]{x} - 3 \log 109 + \log \frac{(109)^3}{\sqrt{4}} = \log \frac{27}{2} + 9 \log 1$.
- (D) $\ln(x+3) - \ln\left(\frac{x}{20} + 1\right) = 2 \ln \sqrt{10}$.
- (E) $\log_x 125 = 3$.

- [10] 3. You are planning to buy a Lexus hybrid four-door sedan two and a half years from now and you will need a \$30000 down payment. The present rate of annual interest is 7.5%.
- (A) If the compounding is weekly, then what is your weekly payment?
- (B) If the compounding is bi-monthly (twice a month), what are your bi-monthly payments?
- (C) How much more of a return does the weekly compounding give over the bi-monthly compounding?
- [10] 4. You buy a \$2500000 “camping and trailer park” with a down payment of \$500000, and you finance the remaining amount with a 10 year mortgage at 6.5% compounded weekly at your favourite bank.
- (A) What are your weekly payments?
- (B) What is your remaining balance after 7 years?
- (C) If you add \$1500 to each of your weekly payments, then how soon will you have paid off your debt?
- (D) How much would you save by using method (C) rather than method (A)?
- [10] 5. A grain company wants to lease a fleet of 20 covered hopper railcars with a combined capacity of 108000 ft³. Hoppers with three different carrying capacities are available: 3000 ft³, 4500 ft³ and 6000 ft³.
- (A) How many of each type of hopper should they lease? (There are no fractional hoppers!)
- (B) The monthly leasing rates are: \$180, \$225 and \$325 for the 3000 ft³, 4500 ft³ and 6000 ft³ hoppers respectively. Which solution in (A) minimizes the monthly leasing?
- [10] 6. By means of Gauss-Jordan Row Reduction find the inverse A^{-1} of the matrix

$$A = \begin{pmatrix} 4 & 5 & 6 \\ 1 & 1 & 1 \\ 4 & 5 & -4 \end{pmatrix}$$

No other method of calculating A^{-1} will be accepted!

- [10] 7. An economy is based on the three sectors of agriculture, energy and manufacturing. Production of one dollar's worth of agriculture requires an input of \$0.2 from agriculture and \$0.1 from energy. Production of one dollar's worth of energy requires an input of \$0.3 from energy and \$0.4 from manufacturing. Production of one dollar's worth of manufacturing requires an input of \$0.4 from agriculture, \$0.1 from energy and \$0.2 from manufacturing.

(A) Write the technological matrix M for this economy.

(B) Write the matrix system that the output vector $X = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$ must satisfy for a final demand of

$$D = \begin{pmatrix} 40 \\ 20 \\ 30 \end{pmatrix} \text{ (in billions of dollars),}$$

where x_1, x_2 , and x_3 are the outputs from agriculture, energy and manufacturing respectively (in billions of dollars).

(C) Find this output vector X . (Hint: Use Gauss-Jordan Row Reduction!)

- [10] 8. Extremize $Q(x, y) = 20x - 40y$ subject to

$$2x + 3y \geq 12, \quad x + 2y \leq 30, \quad 3x + y \leq 30, \quad x \geq 0, \quad y \geq 0.$$

- [10] 9. A 4-person grievance committee is to be selected from departments A and B, consisting of 15 and 20 people respectively. In how many ways can the following committees be selected?

(A) 3 from department A and 1 from department B.

(B) 2 from department A and 2 from department B.

(C) All from department A.

(D) 4 regardless of department.

(E) At least 3 from department A.

- [10] 10. A shipment of 80 wrist watches contains 6 that are defective. The receiving department selects 10 at random for testing, and rejects the whole shipment if one or more are defective.

(A) What is the probability that this shipment shall be accepted?

(B) What is the probability that this shipment shall be rejected?